Degenerate Transportation Problem

In a transportation problem, if a basic feasible solution with m origins and n destinations has less than m +n -1 positive X_{ij} i.e. occupied cells, then the problem is said to be a **degenerate transportation problem**. The degeneracy problem does not cause any serious difficulty, but it can cause computational problem wile determining the optimal minimum solution.

Therefore it is important to identify a degenerate problem as early as beginning and take the necessary action to avoid any computational difficulty. The degeneracy can be identified through the following results:

"In a transportation problem, a degenerate basic feasible solution exists if and only if some partial sum of supply (row) is equal to a partial sum of demand (column). For example the following transportation problem is degenerate. Because in this problem

$a_1 = 400 = b_1$
$a_2 + a_3 = 900 = b_2 + b_3$

	Warehouses			
Plant	w1	w2	w3	**Supply** (a_i)
X	20	17	25	400
Y	10	10	20	500
Unsatisfied demand	0	0	0	400
Demand (b_j)	400	400	500	1300

There is a technique called perturbation, which helps to solve the degenerate problems.

Perturbation Technique:

The degeneracy of the transportation problem can be avoided if we ensure that no partial sum of ai (supply) and bj (demand) is equal.

We set up a new problem where

$a_i = a_i + d$ $\quad$ $i = 1, 2, \ldots\ldots, m$
$b_j = b_j$ $\quad$ $j = 1, 2, \ldots\ldots, n - 1$
$b_n = b_n + m_d$ $\quad$ $d > 0$

This modified problem is constructed in such a way that no partial sum of a_i is equal to the b_j.
Once the problem is solved, we substitute d = 0 leading to optimum solution of the original problem.

Example:

Consider the above problem

Plant	**Warehouses** w_1	w_2	w_3	**Supply** (a_i)
X	20	17	25	400 + d
Y	10	10	20	500 + d
Unsatisfied demand	0	0	0	400 + d
Demand (b_j)	400	400	500+ 3d	1300 + 3d

Now this modified problem can be solved by using any of the three methods viz. North-west Corner, Least Cost, or VAM.

Transportation Problem Maximization

There are certain types of transportation problem where the objective function is to be maximized instead of minimized. These kinds of problems can be solved by converting the maximization problem into minimization problem. The conversion of maximization into minimization is done by subtracting the unit costs from the highest unit cost of the table.

Example

A company has three factories located in three cities viz. X, Y, Z. These factories supplies consignments to four dealers viz. A, B, C and D. The dealers are spread all over the country. The production capacity of these factories is 1000, 700 and 900 units per month respectively. The net return per unit product is given in the following table.

Factory	**Dealers** A	B	C	D	**capacity**
X	6	6	6	4	1000
Y	4	2	4	5	700
Z	5	6	7	8	900
Requirement	900	800	500	400	2600

Determine a suitable allocation to maximize the total return.

This is a maximization problem. Hence first we have to convert this in to minimization problem.

The conversion of maximization into minimization is done by subtracting the unit cost of the table from

the highest unit cost.

Look the table, here 8 is the highest unit cost. So, subtract all the unit cost from the 8, and then we get the revised minimization transportation table, which is given below.

Dealers

Factory	A	B	C	D	**capacity**
X	2	2	2	4	1000 = a1
Y	4	6	4	3	700 =a2
Z	3	2	1	0	900 =a3
Requirement	900=b1	800=b2	500=b3	400=b4	2600

Now we can solve the problem as a minimization problem.

The problem here is degenerate, since the partial sum of $a_1=b_2+b_3$ or $a_3=b_3$. So consider the corresponding perturbed problem, which is shown below.

Dealers

Factory	A	B	C	D	**capacity**
X	2	2	2	4	1000+d
Y	4	6	4	3	700+d
Z	3	2	1	0	900+d
Requirement	900	800	500	400+3d	2600+3d

First we have to find out the basic feasible solution. The basic feasible solution by lest cost method is $x_{11}=100+d$, $x_{22}=700-d$, $x_{23}=2d$, $x_{33}=500-2d$ and $x_{34}=400+3d$.

Once if the basic feasible solution is found, next we have to determine the optimum solution using MODI (Modified Distribution Method) method. By using this method we obtain

$u_1+v_1=2$ $u_1+v_2=2$ $u_2+v_2=6$

$u_2+v_3=4$ $u_3+v_3=1$ $u_3+v_4=0$

Taking $u_1=0$ arbitrarily we obtain

$u_1=0$, $u_2=4$, $u_3=1$ and
$v_1=2$, $v_2=3$, $v_3=0$

On verifying the condition of optimality, we know that

$C_{12}-u_1-v_2 < 0$ and $C_{32}-u_3-v_2 <0$

So, we allocate x_{12}=700-d and make readjustment in some of the other basic variables.

The revised values are:

$x_{11}=200+d$, $x_{12}=800$, $x_{21}=700-d$, $x_{23}=2d$, $x_{33}=500-3d$, and $x_{34}=400+3d$

$u_1+v_1=2$ $u_1+v_2=2$ $u_2+v_1=4$

$u_2+v_3=4$ $u_3+v_3=1$ $u_3+v_4=0$

Taking u1=0 arbitrarily we obtain

$u_1=0$, $u_2=2$, $u_3=-1$

$v_1=2$, $v_2=2$, $v_3=2$, $v_4=1$

Now, the optimality condition is satisfied.

Finally, taking d=0 the optimum solution of the transportation problem is

$X_{11}=200$, $x_{12}=800$, $x_{21}=700$, $x_{33}=500$ and $x_{34}=400$

Thus, the maximum return is:

6*200 + 6*800 + 4*700 + 7*500 + 8*400 = 15500